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Anomaly in Spin-Wave Spectrum of Magnetic Metals

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It is pointed out that in the band theory of magnetism the magnons have frequencies comparable to the Fermi energy. Therefore, in the calculation of the magnon spectrum of iron, nickel, cobalt, etc., it is the time- or frequency-dependent response function of the electrons which is used, and this function—in contrast with the static response function—does not have a "Kohn kink" at $q=2k_F$. But it is found that in the intermediate-coupling regime (exchange force in the range E_F to $\frac{4}{3}E_F$) the magnons have a small range of momenta near $q=k_F$ where their velocities may be zero or negative, and it is shown how this part of the spectrum may be easily calculated, or approximated analytically.

THE present paper draws attention to a possible range of negative magnon velocities in some band-theoretic ferromagnets. The work was initially motivated by an article of Frikkee and Riste,[1] who found what seemed to them evidence for a Kohn anomaly[2] in the spin-wave spectrum of ferromagnetic *3d* transition-series metals. Their experiment consisted of inelastic neutron scattering on an alloy, Co(91%)Fe(9%), the data indicating a break at magnon wave vectors of magnitude approximately

$$q_0 = 0.07\times 2\pi/a. \tag{1}$$

Although the experimenters state that (1) is related to the diameter of the Fermi surface (FS), they do not attempt to extract the size or shape of the latter from the experiment because of various uncertainties and ambiguities. Later work seems to have cleared up some of the difficulties; in private communication to the present author, Frikkee states that a plausible explanation for their results now requires invoking the Fermi wave vectors of the *4s* electrons, and this may well be the case. As we shall discuss below, a simple model of the *3d* magnetic electrons simply cannot lead to a "Kohn kink" in the magnon spectrum (a region of infinite group velocity) but may, under the proper circumstances, predict a small region of zero and negative group velocity of the magnon spectrum. At first, we thought this was an explanation of the above experiment. At the present time, it no longer appears that the two are related. But it would be of interest if such a region of zero and negative group velocity were experimentally discovered in one of the transition ferromagnets, as a senstitive experimental test of some aspects of the band theory of ferromagnetism. We shall find that in the intermediate-coupling regime only (Stoner gap-parameter in the range $0.95E_F<\Delta<1.33E_F$) the magnon spectrum has a maximum at

$$q_1 \lesssim 0.75 k_F\Delta/E_F, \tag{2}$$

as shown in Fig. 1 and a small negative-velocity region thereafter, before merging into the continuum.

Before discussing the band theory of magnetism it is instructive to review the indirect-exchange theory of magnetism, which is valid for the rare-earth metals but *not* for the transition-series metals. There the localized spins (atomic 4-*f* shells) interact via the medium of conduction electrons. The response of the electronic medium to what is essentially a time-independent perturbation (the recoil energy of a magnon in the rare earths is negligible compared to the Fermi energy E_F) is given by the static response function

$$\chi(q) = 1 + \frac{4k_F^2 - q^2}{4k_F q}\ln\left|\frac{2k_F+q}{2k_F-q}\right|, \tag{3}$$

which has infinite slope at precisely $q=2k_F$, otherwise known as a Kohn anomaly. The crystal-structure parameters and umklapp must still be taken into

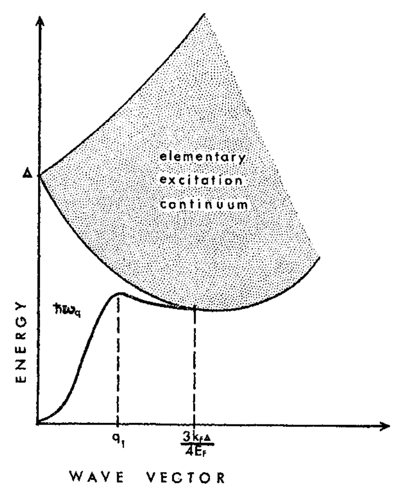

FIG. 1. Schematic plot of magnon energy versus wave vector, indicating maximum (zero velocity) at q_1, very near maximum magnon wave vector at $3k_F\Delta/4E_F$; the separation of these two points is greatly exaggerated in the figure to display negative velocity region. (A more accurate plot for $\Delta=E_F$ is in Fig. 2 of Ref. 8.) The wave vector q_1 is shown in the text to be in the range $0.7\,k_F<q_1<k_F$ for all the values of Δ for which a region of negative velocities is predicted.

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[1] E. Frikkee and T. Riste, *Proceedings of the International Conference on Magnetism, Nottingham, 1964* (Institute of Physics and the Physical Society, London, 1965), pp. 299–301.

[2] W. Kohn, Phys. Rev. Letters **2**, 393 (1959); E. Woll and S. Nettel, Phys. Rev. **123**, 796 (1961).

account before the magnon spectrum can be known,[3] but in the process the branch-cut singularity is preserved. The reader who is more familiar with the original formulation of this theory by Ruderman and Kittel[4] will recall that this anomaly shows up there as the "Friedel oscillations" of the Ruderman-Kittel interaction as a function of the distance between localized spins. Unfortunately, none of these simple indirect-exchange theories of magnetism applies to the iron transition-series elements, for the simple reason that the magnetic electrons are not localized, and indeed have a well-defined band structure, electrical conductivity, specific heat, etc.

To understand the experiment of Ref. 1 it is therefore necessary to appeal to one of the current-band theories of ferromagnetic transition metals. Various extant theories[5] of magnons for this case have recently been reviewed by Thompson[6] and by Herring[7] and while there is no universal agreement among the various authors, nevertheless a common theme may be said to run through these works: the idea that the magnons are propagating, collective oscillations of the medium of magnetic electrons, and that therefore it is the *time-dependent* or *dynamic* response of the electrons which matters. The dynamic-response function which replaces (2) no longer has any vestige of a branch point at $2k_F$, for either of two reasons: (a) in strong-coupled ferromagnets the excitation energies of the individual particles making up the collective mode, and of the wave itself, are all of the order of the Stoner gap parameter Δ which exceeds E_F; therefore the retardation effects can *not* be ignored even in lowest approximation. It is then found (see below) that no vestige of the singularity at $2k_F$ remains. Or, (b) in weak-coupled ferromagnets ($\Delta<E_F$) the magnons only exist over a range of wave vectors $q<0.75\ k_F$, the modes for higher wave vectors being just the usual particle-hole continuum excitations of the Fermi sea (such as indicated by the shaded portion of Fig. 1 but displaced to longer wavelengths). Thus no magnons exist at $2k_F$ and the question of the Kohn anomaly in the weak-coupled ferromagnets does not even arise.

In searching for the region of phase space where magnons have negative velocity we use a simple model previously discussed at some length.[8,9] Earlier, numerical calculations[8] have not indicated the existence of any maximum for $\Delta \lesssim 0.95\ E_F$; for $\Delta \equiv E_F$ they have definitely revealed such a maximum,[6,8] occurring at $q_1=0.72\ k_F$. For Δ in the range

$$(\hbar^2 k_F^2/2m \equiv E_F) \lesssim \Delta \leq \tfrac{4}{3}E_F\,, \tag{4}$$

the maximum occurs very near to q_1 given by Eq. (2) above. For $\Delta>\frac{4}{3}E_F$ there is again no maximum, and the magnon frequency $\omega(q)$ is a monotonically increasing function of q until such point as it merges with the continuum and disappears. The results (2) and (4) taken together imply q_1 *is always less than* k_F. They are obtained by studying the equations for the magnon energy[9]:

$$2k_F q\hbar^2/3m\Delta = L(Q)\,, \tag{5}$$

where

$$Q=k_F q\hbar^2/m\left(\Delta+\frac{\hbar^2q^2}{2m}-\hbar\omega(q)\right), \tag{6}$$

and

$$L(Q)=Q^{-1}-\tfrac{1}{2}(Q^{-2}-1)\ln\left|\frac{1+Q}{1-Q}\right|. \tag{7}$$

These are the equations appropriate to $\Delta \geqslant E_F$. Using (5) and (6) we obtain the magnon energy and velocity in forms more suitable for analysis:

$$\hbar\omega(q)=\Delta\{1-3L(Q)/2Q+9L^2(Q)\Delta/16E_F\}\,, \tag{8}$$

and

$$v(q)=\frac{d\omega}{dq}=\frac{\hbar k_F}{m}\{-Q^{-1}+L/Q^2L'+3\Delta L/4E_F\}\,, \tag{9}$$

where $L'\equiv\partial L/\partial Q$. The point where the magnons merge with the continuum corresponds to $Q=1$, hence $L=1$ by Eq. (7). Insertion into (5) gives $q=0.75\ k_F\Delta/E_F$ as the wave vector, and $\hbar\omega(q)=(9\Delta/16E_F-\frac{1}{2})\Delta$ as the energy of this point. Magnons below the continuum have $Q<1$. We may calculate the negative region of (9) to lowest order in $(1-Q)$, as it will be very close to where the magnons merge with the continuum. Thus $Q\approx L\approx 1$, but $L'\approx \ln(2/1-Q)$. Thus the negative-velocity range is *approximately* given by

$$1>Q>1-2\exp\left[-\left(\frac{4E_F}{4E_F-3\Delta}\right)\right] \tag{10}$$

for any value of the Stoner gap parameter in the range (4). The absence of negative velocities in strong- or weak-coupling means that in the event it is experimentally observed, this phenomenon would approximately determine the coupling constant.

We thank Dr. Frikkee for illuminating correspondence.

[3] D. Mattis and W. Donath, Phys. Rev. **128**, 1618 (1962); see also Ref. 9, pp. 198–207 and Appendix.

[4] M. A. Ruderman and C. Kittel, Phys. Rev. **96**, 99 (1954).

[5] A partial bibliography might include: T. Izuyama, Progr. Theoret. Phys. (Kyoto) **23**, 969 (1960); T. Ruijgrok, Physica **28**, 877 (1962); M. M. Antonoff, Bull. Am. Phys. Soc. **8**, 227 (1963); E. D. Thompson, Ann. Phys. (N.Y.) **22**, 309 (1963); T. Nakamura, Phys. Rev. Letters **12**, 279 (1964); T. Izuyama, Phys. Rev. Letters **12**, 585 (1964); also Refs. 8, 9.

[6] E. D. Thompson, Advan. Phys. **14**, 213 (1965).

[7] C. Herring, forthcoming in *Magnetism*, edited by G. Rado and H. Suhl (Academic Press Inc., New York, 1966), Vol. 4.

[8] D. Mattis, Phys. Rev. **132**, 2521 (1963). Because of an oversight, the calculations in this paper are applicable only to weak-coupling $\Delta \leq E_F$ (for $\Delta>E_F$, see the subsequent correct discussion in D. Mattis, Bull. Am. Phys. Soc. **9**, 559 (1964), and also Ref. 9, pp. 211–218).

[9] D. Mattis, in *The Theory of Magnetism* (Harper and Row, New York, 1965), p. 215.